Wiley
Discrete Dynamics in Nature and Society
Volume 2025, Article ID 3370833, 13 pages
https://doi.org/10.1155/ddns/3370833

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Research Article

Designing Team Projects for Envy-Free Group Collaboration to Overcome Free-Rider Problem

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Received 29 October 2024; Accepted 4 February 2025

Academic Editor: Mengxin Chen

We propose an envy-free team project called "color team project". The primary motivation behind this approach is to prevent free-rider behavior and create a fair evaluation system that avoids jealousy among team members. In the team project, each team member indicates their contribution to the final team output using a color or their name. To evaluate the color team project, we use the number of pixels as the decision matrix, which includes pixels from the entire work ("All"), the methodology section ("Methodology"), the experimental section ("Results"), and the "Title". The attribute weight is determined through steps that include standardization and information entropy. We then determine the ranking order of a team project using either the Simple Additive Weighting (SAW) method or the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method, and it is verified by the analytic hierarchy process (AHP) method. By applying the color team project, we can overcome the free-rider problem and maintain the positive aspects of team projects, such as effective communication, collaboration, and negotiation.

JEL Classification: D23, D71, D83

Keywords: color team project; free-rider problem; pixel; ranking order; team project

1. Introduction

Team project or project-based learning can provide participants with opportunities for effective collaboration, communication, and negotiation. Teamwork is increasingly important in many organizations because of its added value through the collective use of diversified knowledge, skills, and abilities of participants [1]. It is widely known for the positive effects of peer-led team learning in college basic science subjects [2]. However, when the participation and contributions of all team members cannot be guaranteed, team-based assessments can be unfair and inaccurate [3]. In addition, decision makers often lack sufficient information to accurately evaluate organization members [4]. Furthermore, given the self-assembling and fluid nature of collaborative teams in science [5], tested the interdependence of collaborative teams in the same network. In a study by [6], the authors studied the relationship between team size and research performance in the context of cross-disciplinary research. In another study [7], various characteristics and roles of researchers who occupied important positions in collaborative networks were studied. Previous studies usually focused on the impact of external factors on team performance, such as innovation capability, organizational culture, competitive environment, and readiness for change [8–12], while overlooking the individual contributions of team members internally, which could lead to free-riding behavior.

In team collaboration, free-riding refers to the phenomenon where certain members enjoy the benefits of team achievements or shared resources without making corresponding efforts or contributions [13]. This behavior often results in other members shouldering more workload, thereby reducing overall team efficiency and fairness [14]. Minimizing the free-rider problem is important to develop a positive influence toward team project; otherwise, team projects can give a negative attitude to participating students [1]. At the same time, due to the free-rider problem, members with higher abilities or lower costs are more likely to perform worse than those with lower abilities [15]. One of the solutions to the free-rider problem is peer evaluation [16]; Srid-haran et al., 2018). In a study by [17], the authors proposed a theoretical model of free-riding and developed an online assessment system for individual scores (OASIS) to minimize free-riding. Computer-supported collaborative learning (CSCL) was presented to overcome free-riding issues, which is composed of four steps: self-reflection, peer assessment, group assessment, and individual assessment [18]. In another study [19], the authors reported a survey after implementing an online tool, the self and peer assessment resource kit (SPARK), which allows students to evaluate their own and peers' contributions. A simple method was introduced to fire the free-rider [20]. It eliminates the fundamental problems; however, the downside is that it cannot solve the underlying problem. The results in [21] indicate that peer assessment is effective for students who find appropriate evaluation criteria in reducing free-riders. In a study by [22], the authors stressed that appropriate training on effective peer feedback is essential for the successful self and peer assessment process. These findings are supported by numerous studies that highlight the effectiveness of peer evaluation. Meanwhile, other studies have explored approaches such as punishment measures [23, 24]) and shared interests [25–27] to tackle the issue of free-riding.

To overcome the free-rider problem, we propose the use of an envy-free team project called the "color team project". Here, the envy-free team refers to a fair and transparent team structure where each member's contributions are justly recognized, without causing jealousy or dissatisfaction due to the contributions of others. In the color team project, each team member indicates their contribution to the final team output by color or individual's name. By applying the color team project, we can preserve the merits of the team project, such as communication, collaboration, and negotiation. We construct a decision matrix for the contribution of research group members and further adopt various statistical methods to ultimately determine the ranking ratio of the team project. The computational results of the model can objectively reflect the degree of work contribution of the research team members. The proposed color team project can effectively prevent the free-rider problem and enhance the benefits of team projects.

The novelty of this paper lies in the introduction of the "color team project" concept, using pixel counts as a decision matrix. Compared to previous studies, this paper emphasizes the combination of image processing techniques with multiple decision methods, providing a more objective and accurate evaluation of team contributions, overcoming the subjectivity and bias commonly found in traditional methods. Using Algorithm 1, we can obtain extract effective pixels from an image.

The contributions of this study are as follows:

1. Objective Evaluation Mechanism: A color-based quantitative method is proposed, integrating pixel statistics and decision-making approaches to establish a fair and transparent contribution assessment mechanism.
2. Envy-Free Collaboration: The quantitative evaluation ensures fair recognition of each member's contribution, avoiding dissatisfaction or jealousy, and enhancing team efficiency.
3. Broad Applicability: The proposed framework is not limited to research teams but can be applied to education, corporate task management, and healthcare collaboration scenarios.

The layout of this paper is organized as follows. In Section 2, we introduce the concept of the color team project. In Section 3, we propose an automatic scoring system for the members in the team project. Section 4 is the discussion section. In Section 5, we present our conclusion.

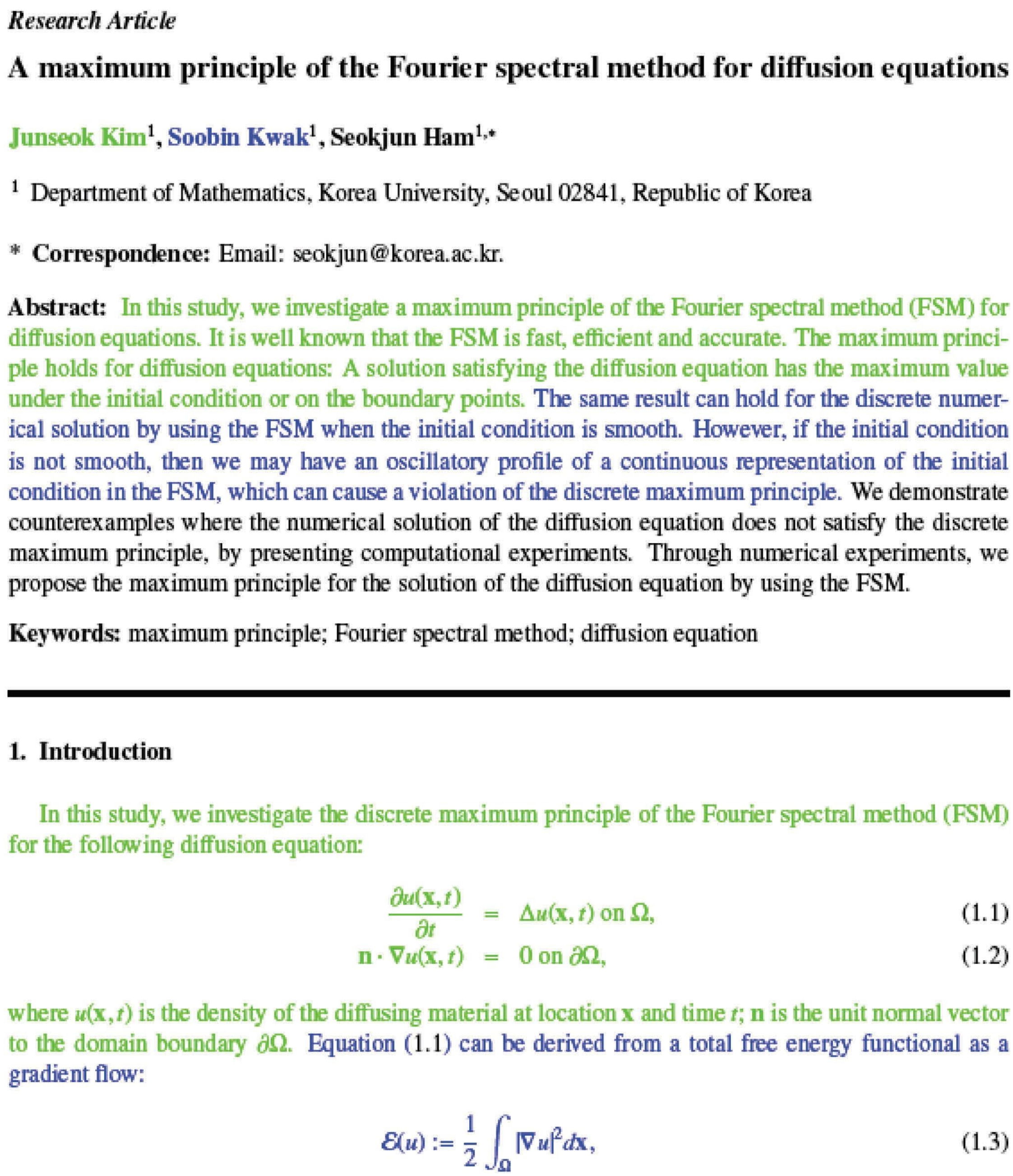
2. Color Team Project

In the color team project we propose, each team member represents their contribution to the final team output using a unique color, as shown in Figures 1 and 2. Each team member chooses a unique color and represents their name using that color. Figure 1 shows the first page of the final report from a color team project. Each team member expresses their contribution by coloring the contributed sentences with different colors, such as black, blue, or green.

Figure 2 displays four slides from the final PowerPoint (PPT) presentation of a color team project. Each team member indicates their contribution to the PPT through the border color.

3. Automatic Scoring System

This section presents a quantitative evaluation of the contribution of each team member to the team project by calculating the number of pixels on the color page that corresponds to their contribution. As shown in Figure 3, four members have jointly completed the paper in the figure, the contributions of each member are represented using distinct colors. The authors are named by A, B, C, and D,



Research Article

A maximum principle of the Fourier spectral method for diffusion equations

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Abstract: In this study, we investigate a maximum principle of the Fourier spectral method (FSM) for diffusion equations. It is well known that the FSM is fast, efficient and accurate. The maximum principle holds for diffusion equations: A solution satisfying the diffusion equation has the maximum value under the initial condition or on the boundary points. The same result can hold for the discrete numerical solution by using the FSM when the initial condition is smooth. However, if the initial condition is not smooth, then we may have an oscillatory profile of a continuous representation of the initial condition in the FSM, which can cause a violation of the discrete maximum principle. We demonstrate counterexamples where the numerical solution of the diffusion equation does not satisfy the discrete maximum principle, by presenting computational experiments. Through numerical experiments, we propose the maximum principle for the solution of the diffusion equation by using the FSM.

Keywords: maximum principle; Fourier spectral method; diffusion equation

1. Introduction

In this study, we investigate the discrete maximum principle of the Fourier spectral method (FSM) for the following diffusion equation:

$$\frac{\partial u(\mathbf{x},t)}{\partial t} = \Delta u(\mathbf{x},t) \text{ on } \Omega, \quad (1.1)$$

$$\mathbf{n}\cdot\nabla u(\mathbf{x},t) = 0 \text{ on } \partial\Omega, \quad (1.2)$$

where $u(\mathbf{x},t)$ is the density of the diffusing material at location $\mathbf{x}$ and time t; $\mathbf{n}$ is the unit normal vector to the domain boundary $\partial\Omega$. Equation (1.1) can be derived from a total free energy functional as a gradient flow:

$$\mathcal{E}(u) := \frac{1}{2}\int_\Omega |\nabla u|^2 d\mathbf{x}, \quad (1.3)$$

Figure 1: Example of a color team project: paper. Modified from [28].

whose titles are Ph.D., Master, Master, and Supervisor. We assign appropriate values to the title attribute for each member in the team project. Usually, the work of a master's degree in a research group also requires the assistance of a Ph.D., therefore, the title attribute value of a Ph.D. is also higher than that of a master's degree. In addition, the supervisor is usually a corresponding author, a contributor to idea, and guides the whole topic, whose title attribute value should be high. The value of the title attribute should be assigned according to the specific situation of the research group members. In this study, we set the values of the title attribute as 2, 1, 1, and 5, corresponding to A, B, C, and D, respectively.

Furthermore, we calculate the total pixel count attributed to members A, B, C, and D throughout the entire paper by converting the paper into a digital image format. As shown in Figure 4, the grid size of the image is 774×466, resulting in a total pixel count of 360,684. The areas corresponding to each team member's contribution are marked with different colors, and the pixel count within each member's designated area is calculated. We calculate the number of pixels attributed to authors A, B, C, and D and list them in Table 1.

We consider the numerical part in Table 1 as a two-dimensional matrix $(d_{i,j})_{m\times n}$, where i indicates the member and j represents the attribute. Here, $m = n = 4$. Based on the decision attribute matrix, we adopt the Simple Additive

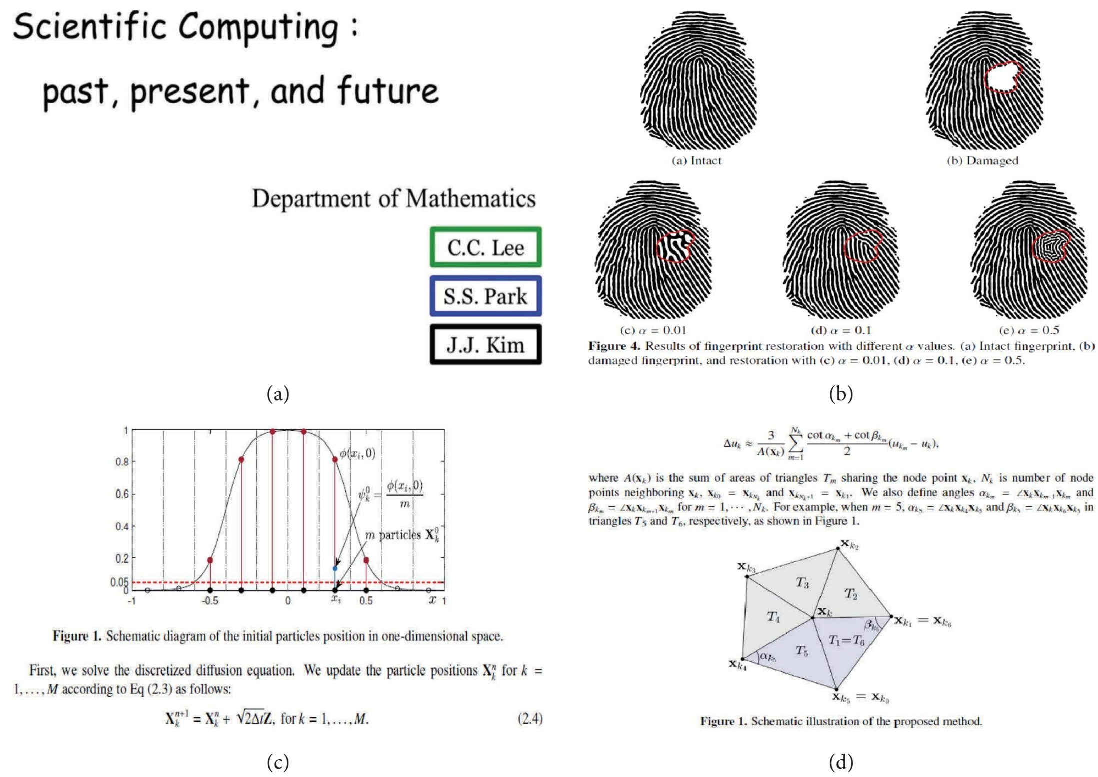


Figure 2: Example of a color team project: PowerPoint presentation. (a) The first page of the PPT. (b)–(d) Modified from [29–31]), respectively.

Weighting (SAW) method to determine the ranking order of the members in the team project. We first standardize the scaling transformation as follows:

$$r = \left(r_{i,j}\right), \qquad r_{i,j} = \frac{d_{i,j}}{\sum_{i=1}^{m} d_{i,j}}, \tag{1}$$

where the column sum of r is 1. The computational results are listed in Table 2. Furthermore, the decision attribute matrix can be also standardized by the following scaling transformation, where column sum of r'^2 is 1.

$$r' = \left(r'_{i,j}\right), \qquad r'_{i,j} = \frac{d_{i,j}}{\sqrt{\sum_{i=1}^{m} d_{i,j}^2}}. \tag{2}$$

Subsequently, we determine the attribute weights, which represent the importance of each attribute to the decision objective. We adopt the following information entropy method [32].

$$\text{Entropy}_j = -\frac{1}{\ln m}\sum_{i=1}^{m}\left(r_{i,j}\times \ln r_{i,j}\right). \tag{3}$$

The smaller Entropy_j, the better the corresponding attribute can distinguish between advantages and disadvantages. Thus, we set $1 - \text{Entropy}_j$ to replace it, and utilize the following equation to normalize the discrimination.

$$\text{Weight}_j = \frac{1 - \text{Entropy}_j}{\sum_{j=1}^{n}\left(1 - \text{Entropy}_j\right)}. \tag{4}$$

Finally, we use the SAW method to determine the ranking score of each member in the team project. The computational results are listed in Table 2. We observe that the ranking order of four members is: A-D-B-C. D is not high in the proportion of pixels. However, D is a supervisor, whose title attribute value is high, which can improve its contribution in the paper, and it can be placed last as a corresponding author. The ranking of the remaining three members is based on their corresponding workload. In addition, the weights of the four attributes are: "Title", "Methodology", "Results", "All". This is also consistent with our subjective impression.

$$\text{Score}_i = \sum_{j=1}^{n}\left(\text{Weight}_j \times r_{i,j}\right). \tag{5}$$

Next, we use analytic hierarchy process (AHP) method to verify the above results. AHP was proposed in the early 1970s to solve decision-making problems in individuals and groups [33]. AHP represents decision problems through a hierarchy and prioritizes alternatives based on the decision maker's judgment of the entire system. The primary merit of AHP is to convert the human judgments into the comparisons of the importance between the two of several factors by making a hierarchical structure to convert the qualitative judgments, which are difficult to quantify into an operational comparison of the importance. In this paper, we built

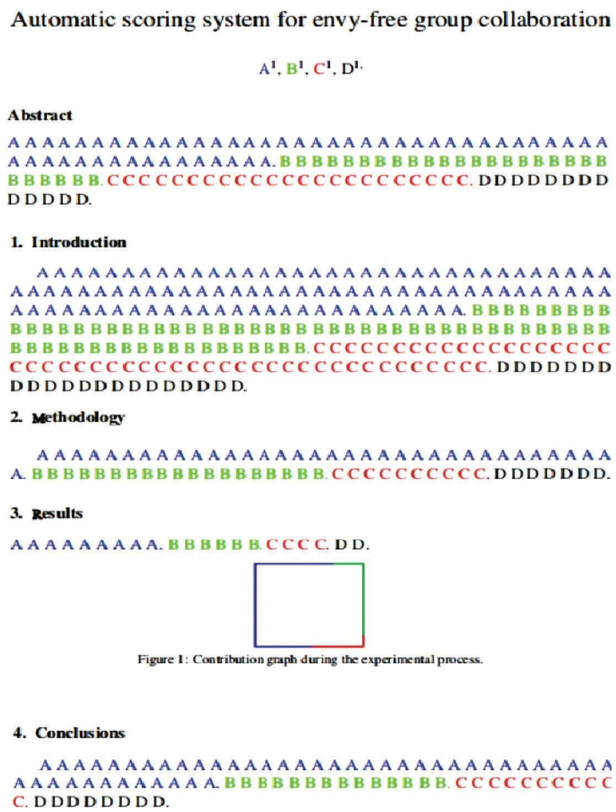


Figure 3: Example of automatic scoring system for a color team project.

Table 1: The members in team project and their decision attributes.

Member	All	Methodology	Result	Title
A	15,499	2163	1366	2
B	10,478	1499	882	1
C	6853	745	513	1
D	4459	733	325	5

three layers, as shown in Figure 5. The Objective layer is the influence on team project ordering, the Criterion layer contains “All”, “Methodology”, “Results”, and “Title”, and the Plan layer represents authors A, B, C, and D.

Then, using the data given in Table 1, we perform a paired comparison to assess the relative importance of the elements [33]. The paired comparison relies on the decision maker’s subjective judgment of the relative importance between two factors, using a scale of 1 to 9 for assignment. A value of one indicates that the two factors are equally important, while a value of 9 signifies that one factor is absolutely more important than the other. For example, if factor “P” is regarded significantly more important than factor “Q”, the weight assigned to “P” relative to “Q” would be 5. Conversely, the weight of “Q” relative to “P” is automatically assigned as the reciprocal, i.e., 1/5.

The AHP method uses heap comparisons at each level of the hierarchy to determine the comprehensive weight of the decision elements. Specifically, we first construct judgment matrices for the four elements: “All,” “Methodology,” “Results,” and “Title,” as shown in Tables 3, 4, 5, and 6. Next, we process the matrices using the geometric mean method and calculate the relative weights of each decision element by normalizing the eigenvectors. Furthermore, to ensure the validity of the judgment matrices, we compute the consistency ratio for each matrix, all of which pass the consistency test. Table 7 shows the weights of the factors that affect the ranking of team projects. Here, the weights of the target layer use the results in Table 2. Notably, the ranking order of the four members is A-D-B-C. It is consistent with the ranking of each member’s contribution degree in the team project by using SAW method, which can further demonstrate the reliability of the proposed SAW method in calculating each member’s contribution to the team project.

Furthermore, we also adopt the TOPSIS method to reflect the score of each member. The calculation process is described as follows:

$$v = \left(v_{i,j}\right),\ v_{i,j} = \text{Weight}_j \times r'_{i,j}. \tag{6}$$

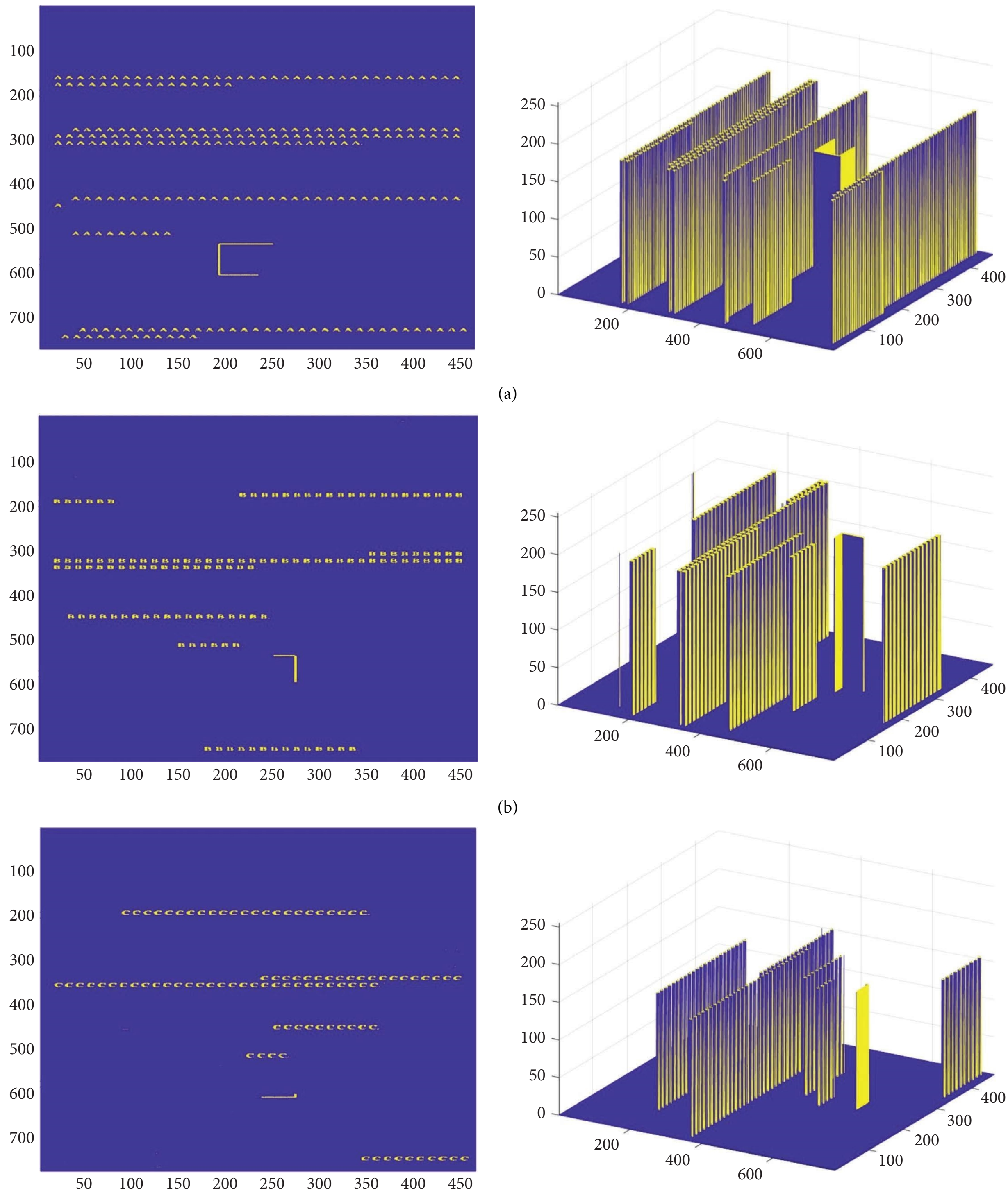
The scaling transformation of r' is presented in Table 8. Then, we find the positive and negative ideal solutions, that is, the vector consisting of the largest element and the smallest element, which are represented by v^+ and v^-. Afterward, we compute

$$\begin{aligned} S_i^+ &= \sqrt{\sum_{j=1}^{n}\left(v_{i,j} - v_j^+\right)^2}, \\ S_i^- &= \sqrt{\sum_{j=1}^{n}\left(v_{i,j} - v_j^-\right)^2}, \end{aligned} \tag{7}$$

$$\begin{aligned} Q_i^+ &= \frac{S_i^-}{S_i^+ - S_i^-}, \\ \text{Score}'_i &= \frac{Q_i^+}{\sum_{i=1}^{m} Q_i^+}. \end{aligned} \tag{8}$$

Table 9 presents the ranking scores of the TOPSIS method, where the four members are ranked as D-A-B-C. In addition, Figure 6 provides a visualization of the scores for each member across different methods. It can be observed that, except for member D, the ranking results for other members are largely consistent. In the TOPSIS method, the weight proportion of the title attribute is higher, which leads to a greater contribution from member D.

To demonstrate the impact of different weights of attributes on the results, we use the TOPSIS method to continue exploring whether the ranking of paper authors changes when attribute values change. If the supervisor’s guidance is limited and the contribution is small, the value of the title attribute D can be reduced, and the title attributes of A, B, C, and D can be changed to (2, 1, 1, 2); While the supervisor’s contribution is significant, the title attribute of A, B, C, and D can be changed to (2, 1, 1, 10). The

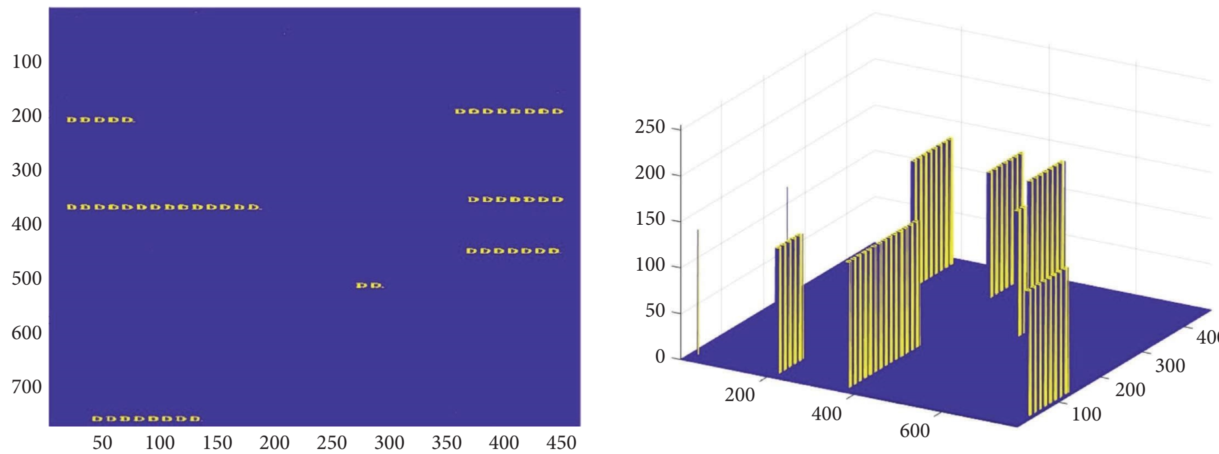


(a)

(b)

(c)

Figure 4: Continued.



(d)

Figure 4: Pixel count contributed by the corresponding author.

Table 2: Results of the ranking score of the SAW method.

Member	All	Methodology	Result	Title	Score
A	0.4156	0.4208	0.4426	0.2222	0.3429
B	0.2810	0.2917	0.2858	0.1111	0.2138
C	0.1838	0.1449	0.1663	0.1111	0.1426
D	0.1196	0.1426	0.1053	0.5555	0.3007
Entropy	0.9283	0.9243	0.9046	0.8289	—
1-entropy	0.0717	0.0757	0.0954	0.1711	—
Weight	0.1732	0.1829	0.2305	0.4134	—

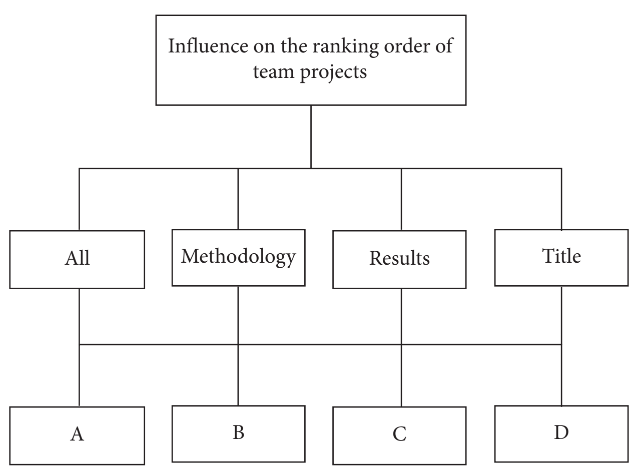


Figure 5: Structure hierarchy.

Table 3: Judgment matrix based on "All".

Number	A	B	C	D
A	1	2	3	4
B	1/2	1	2	3
C	1/3	1/2	1	2
D	1/4	1/3	1/2	1

corresponding computation scores of each member are listed in Table 10, with the rankings visually represented in Figure 7. The results clearly imply the significant of the corresponding author. If the supervisor plays a low role, the corresponding ranking will be reduced, and D should not be the corresponding author. Conversely, if the supervisor's

Table 4: Judgment matrix based on "Methodology".

Number	A	B	C	D
A	1	2	3	3
B	1/2	1	2	3
C	1/3	1/2	1	1
D	1/3	1/2	1	1

Table 5: Judgment matrix based on "Results".

Number	A	B	C	D
A	1	2	3	4
B	1/2	1	2	3
C	1/3	1/2	1	2
D	1/4	1/3	1/2	1

Table 6: Judgment matrix based on "Title".

Number	A	B	C	D
A	1	2	2	1/3
B	1/2	1	1	1/5
C	1/2	1	1	1/5
D	3	5	5	1

Table 7: Results of the ranking score of the AHP method.

Member	All	Methodology	Result	Title	Score
A	0.4673	0.4554	0.4673	0.2090	0.3583
B	0.2772	0.2628	0.2772	0.1093	0.2051
C	0.1601	0.1409	0.1691	0.1093	0.1376
D	0.0954	0.1409	0.0954	0.5725	0.3010
Weight	0.1732	0.1829	0.2305	0.4134	—

Table 8: Scaling transformation of r'.

Member	All	Methodology	Result	Title
A	0.7591	0.5679	0.3853	0.1942
B	0.5131	0.6561	0.8130	0.0971
C	0.3357	0.4582	0.4268	0.0971
D	0.2184	0.1925	0.0917	0.9713

Table 9: Results of the ranking score of the TOPSIS method.

	S^+	S^-	Q	Score$'$
A	0.4520	0.1117	0.4790	0.3365
B	0.4976	0.1712	0.2488	0.1748
C	0.5070	0.0804	0.0868	0.0610
D	0.1790	0.4968	0.6088	0.4277

contribution is significant, the final calculated ranking score will be further improved, however, the ranking of the other three authors remains unchanged.

Similarly, we show the impact of the attributes of "Methodology" and "Results". As shown in Table 11 and Figure 8, when the pixel proportion of C in "Methodology" and "Results" increases, the author order of B and C should be reversed. The order of the members except the corresponding author is A-C-B. Considering the process of writing a paper, the methodology and experimental parts require more time and effort, especially the experimental part, which requires a lot of time.

Then, we increase the pixel density of author B. The calculation results, presented in Table 12 and Figure 9, imply that B may be considered as the first author of this paper. Even though B's pixel ratio in the whole paper is not as high as A's, and even the title of B is a master and A is a Ph.D., nevertheless, B has completed most of the methodological

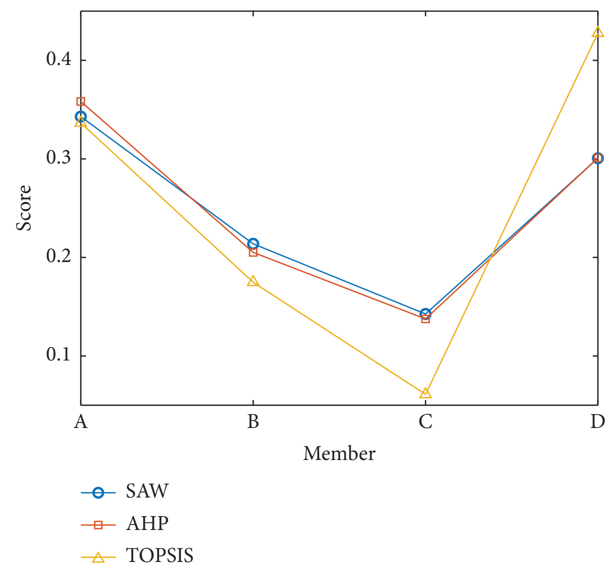


Figure 6: Scores of each member across different methods.

Table 10: Ranking score when "Title" attribute changes.

	Title	A	B	C	D
Score	(2, 1, 1, 2)	0.5477	0.2864	0.0889	0.0770
	(2, 1, 1, 5)	0.3365	0.1748	0.0610	0.4277
	(2, 1, 1, 10)	0.1995	0.1003	0.0338	0.6665

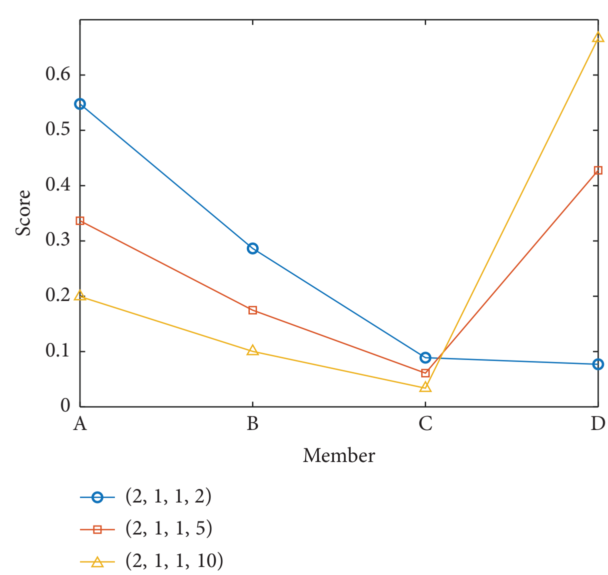


Figure 7: Visualization of the ranking scores as the "Title" attribute changes.

Table 11: Ranking score when pixel proportion of C in "Methodology" and "Results" increases.

Member	All	Methodology	Result	Title	Score-original	Score-new1
A	15,499	2163	1366	2	0.3365	0.2692
B	10,478	1499	882	1	0.1748	0.1292
C	6853	1745	1513	1	0.0610	0.1779
D	4459	733	325	5	0.4277	0.4237

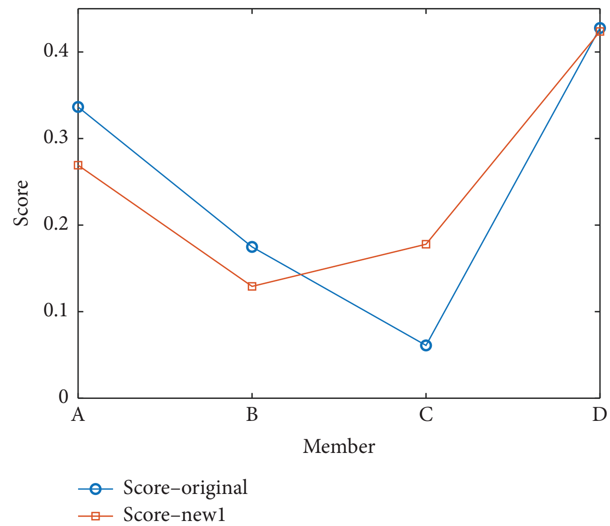


Figure 8: Visualization of the ranking scores when pixel proportion of C in "Methodology" and "Results" increases.

Table 12: Ranking score when pixel proportion of B in "Methodology" and "Results" increases.

Member	All	Methodology	Result	Title	Score-New1	Score-new2
A	15,499	2163	1366	2	0.2692	0.2333
B	10,478	2499	2882	1	0.1292	0.2829
C	6853	1745	1513	1	0.1779	0.1629
D	4459	733	325	5	0.4237	0.3209

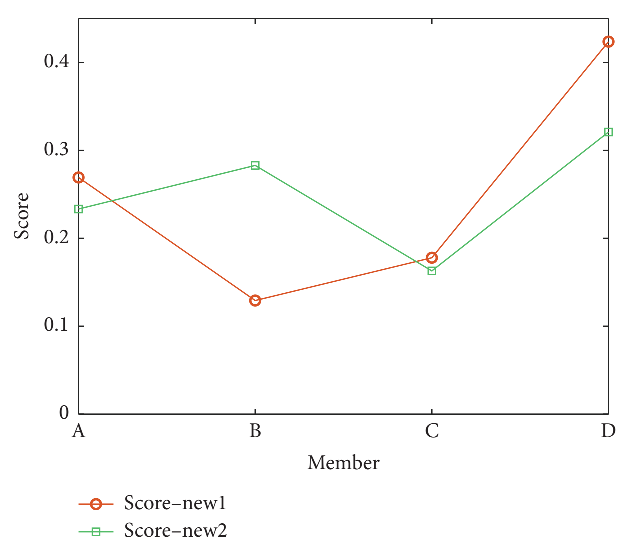


Figure 9: Visualization of the ranking scores when pixel proportion of B in "Methodology" and "Results" increases.

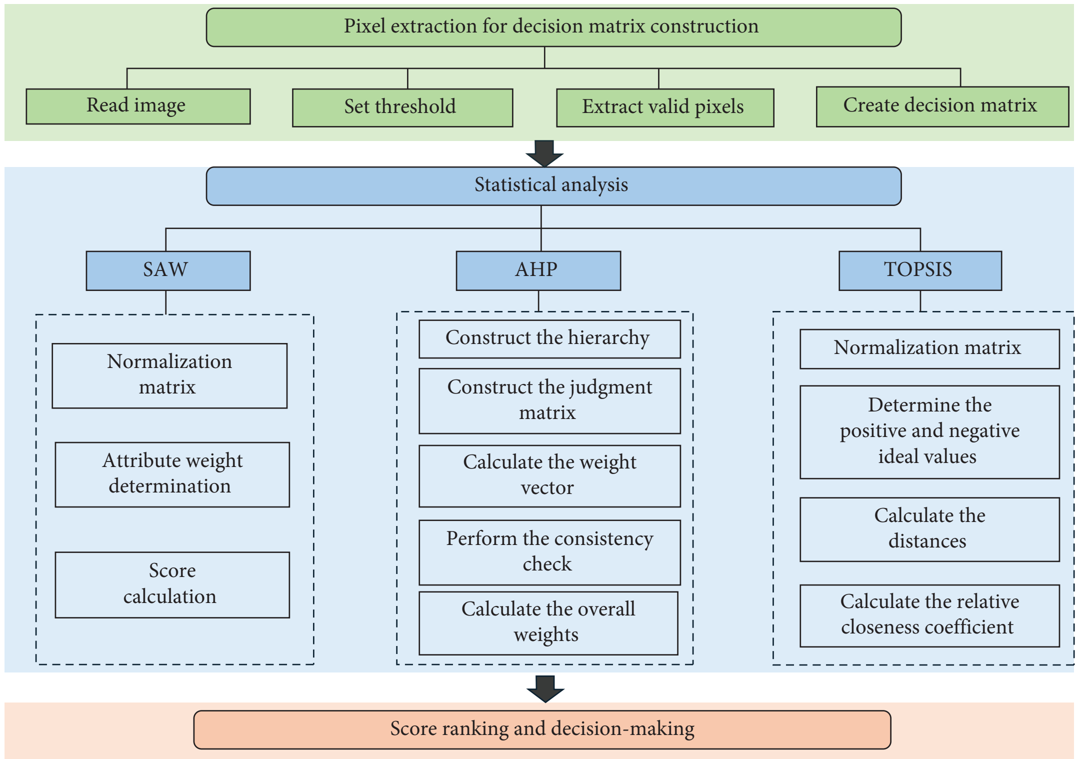


Figure 10: Overall frame diagram.

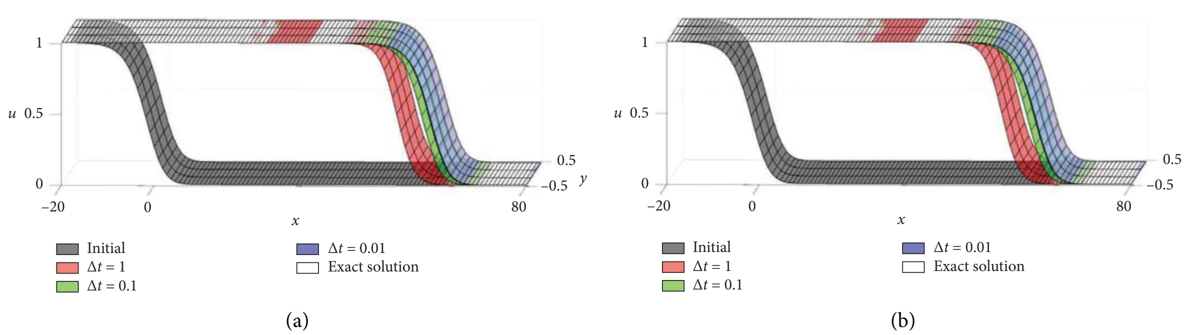


Figure 11: Examples of a joint contribution in PowerPoint of a color team project. Modified from [34].

construction and experimental implementation of this paper, which helps B achieves the highest ranking score. Then, the appropriate sorting should be B-A-C.

To promote understanding, we provide the overall flowchart in Figure 10 and pseudocode in the Appendix.

4. Discussion

The team project is a classical method in which several individuals form a team and perform a difficult task, which is difficult for an individual to do alone, together. Although team projects require a lot of effort from learners, unlike achieving through simple learning alone, participants can naturally develop communication skills in the process of working together toward a common goal. However, there may be an issue with social loafing among some participants. When carrying out a project, it is often necessary to assign to each team member through the allocation of detailed roles. In particular, when there is a lack of trust between team members, there is a tendency to divide tasks specifically to minimize the probability of a free-rider problem. The free-rider problem is a common issue in team project implementation, where irresponsible participants do not contribute to the team project and place additional burdens on other participants. To address this problem, self and peer evaluations have been studied actively as a means of penalizing participants who do not actively fulfill their roles, as well as compensating participants who have suffered from free-riders by assigning them additional works.

Team collaboration is inherently dynamic and complex, characterized by the allocation of tasks, contributions, and shared responsibilities among members. These interactions can be effectively modeled as a discrete dynamic system. We have proposed a color team project method, which is expected to encourage more natural participation compared to self or peer evaluation methods. For self or peer evaluation methods to be effective, it is essential that all participants are trained in how to give feedback objectively and exclude personal feelings. The main advantage of the proposed color team project method is its simplicity and efficiency. The proposed method can effectively prevent the free-rider problem because the free-riders must demonstrate their contributions to the team project in an explicit manner.

It should be noted that if multiple participants work on certain parts of the project, then they can demonstrate their contribution through fractions. This can be expressed in a PPT presentation by showing the quantity proportional to each participant's effort, as shown in Figure 11. For instance, Figure 11(a) depicts an example when two participants equally contribute, while (b) illustrates a case where three participants contribute 50%, 25%, and 25%, respectively.

A study conducted by [35] found that there are correlations between team interaction, team efficacy, and psychological safety with team creativity in the context of team project-based learning. By employing the proposed color team project method, it is expected to prevent free-rider problems and encourage students to participate in team projects collaboratively rather than individually.

Our automatic ranking system considers multiple factors in our model, including various components of the research topic, such as method construction, experimental implementation, and positions. These components play crucial roles in the research topic and need to be evaluated separately in the decision-making system. All attribute values, except for positions, can be determined by calculating the number of pixels, which is an advantage of our proposed color team project method.

The title attribute is also considered in our model, as a project is typically completed by researchers of various categories, including Ph.D. students, master's students, and supervisor. The purpose of this consideration is that the supervisor usually guides the research paper on the whole topic. Even if the pixel proportion is not high, its contribution is also outstanding, so the title attribute column is indispensable in our model. Additionally, the title attribute value of a doctor is higher than that of a master's student, given that the work of a master's student in a research group typically requires the assistance of a Ph.D. student.

However, if any author makes significant contributions to the methodology and experimental stages, their ranking will improve under our model's calculations, even if their title attribute value is low. This is consistent with reality, and our proposed method is a practical envy-free group collaboration model.

Although this study focuses on a specific research paper writing project, the proposed methodology is broadly applicable to various collaborative environments, including corporate task management, educational group projects, and software development. These scenarios often involve dynamic processes that require balancing fairness, efficiency, and shared responsibility. Due to the flexibility of the approach, it is also applicable to teams of different sizes and complexities. However, for large teams, we recognize that the complexity of the project may significantly increase. To overcome this challenge, more detailed contribution marking or finer adjustments to evaluation precision can be implemented. By dividing the project into subtasks and evaluating contributions using the proposed method, team dynamics and coordination can be managed effectively. Overall, it provides a practical solution for evaluating and optimizing these dynamics, ensuring smooth project progression.

5. Conclusion

In this paper, we proposed an envy-free team project, which is named as color team project. In the team project, each team member indicates his or her contribution to the final team output by color or individual's name. By applying the color team project, we can preserve the merits of the team project such as communication, collaboration, and negotiation. We used various statistical methods to evaluate the contribution of research team members, and quantitatively scored each member. Through extensive experimental calculations, we have verified the fit of the model in practice, and the results showed that our proposed attributes such as "All", "Methodology", "Results", and "Title" are reasonable, contributing a good approach to managing an envy-free team project.

Appendix A: Pseudocode

Extract effective pixels from an image.

Input: Image matrix B (size $m \times n$), threshold tol
Output: Effective pixels count le

```
B_1 ⟵ Create a zero matrix of size m × n;
for i = 1 to m do
    for j = 1 to n do
        if B(i, j) < tol
            then B_1(i, j) = B(i, j)
            else B_1(i, j) = 0
    end
end
Flatten B_1 into a 1D array (B_11);
Calculate le as the number of non-zero elements in B_11
```

ALGORITHM 1: Extract effective pixels from an image.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no conflicts of interest.

Funding

The corresponding author (Junseok Kim) was supported by the Brain Korea 21 FOUR from the Ministry of Education of Korea.

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